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Periodic bouncing solutions of the Lazer–Solimini equation with weak repulsive singularity

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ABSTRACT

We prove the existence and multiplicity of periodic solutions of bouncing type for a second-order differential equation with a weak repulsive singularity. Such solutions can be cataloged according to the minimal period and the number of elastic collisions with the singularity in each period. The proof relies on the Poincaré–Birkhoff Theorem.

1. Introduction

Differential equations with singularities appear as mathematical models in many scientific areas and have been studied from many viewpoints [1]. In this paper, we consider the singular second order differential equation

$$\ddot{u} - \frac{1}{u^{\alpha}} = p(t), \ u > 0, \tag{1}$$

with parameter $\alpha > 0$ and $p : \mathbb{R} \to \mathbb{R}$ a continuous and 2π-periodic function. In a seminal paper, Lazer and Solimini [2] proved that when $\alpha \geqslant 1$ Eq. (1) has a positive periodic solution if and only if p has negative mean value. The authors also showed that the statement is sharp with respect to the parameter α in the sense that if $0 < \alpha < 1$, a function p with negative mean value can be constructed in such a way that (1) has no periodic solutions. Later, [3, Example 3.9] provided an effective sufficient condition over p for the existence of a classical periodic solution in the weak repulsive case. The particular case $\alpha = 1/2$ has been studied in [4] showing that the equation corresponds to a perturbed isochronous oscillator and resonance conditions on the forcing term $p(t)$ are given.

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