Proceedings of the Royal Society of Edinburgh, page 1 of 11
DOI:10.1017/prm.2021.72

Non-bifurcation of critical periods from semi-hyperbolic polycycles of quadratic centres

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(Received 29 January 2021; accepted 1 November 2021)

In this paper we consider the unfolding of saddle-node

$$X = \frac{1}{xU_a(x,y)}\Big(x(x^\mu - \varepsilon)\partial_x - V_a(x)y\partial_y\Big),$$

parametrized by (ε, a) with $\varepsilon \approx 0$ and a in an open subset A of $\mathbb{R}^\alpha$, and we study the Dulac time $\mathcal{T}(s;\varepsilon, a)$ of one of its hyperbolic sectors. We prove (theorem 1.1) that the derivative $\partial_s\mathcal{T}(s;\varepsilon, a)$ tends to $-\infty$ as $(s, \varepsilon) \to (0^+, 0)$ uniformly on compact subsets of A. This result is addressed to study the bifurcation of critical periods in the Loud's family of quadratic centres. In this regard we show (theorem 1.2) that no bifurcation occurs from certain semi-hyperbolic polycycles.

Keywords: Period function; saddle-node unfolding; Dulac time; asymptotic expansions

2020 *Mathematics subject classification:* 34C07; 34C20; 34C23

1. Introduction and main results

The present paper deals with planar polynomial ordinary differential systems and we study the qualitative properties of the period function of centres. A singular point of a planar differential system is a *centre* if it has a punctured neighbourhood that consists entirely of periodic orbits surrounding it. The largest neighbourhood with this property is called the *period annulus* of the centre and we denote it