Critical periods in planar polynomial centers near a maximum number of cusps

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Abstract

We provide the best lower bound for the number of critical periods of planar polynomial centers known up to now. The new lower bound is obtained in the Hamiltonian class and considering a single period annulus. This lower bound doubles the previous one from the literature, and we end up with at least $n^2 - 2$ (resp. $n^2 - 2n - 1$) critical periods for planar polynomial systems of odd (resp. even) degree n. Key idea is the perturbation of a vector field with many cusp equilibria, whose construction is by itself a nontrivial construction that uses elements of catastrophe theory.
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1. Introduction

The study of the number of oscillations of the period function associated to a planar polynomial center has a long history that took a start with the seminal paper by Chicone and Jacobs in 1989 where the authors did a bifurcation analysis of the critical periods for planar vector fields in [2]. In classical Liénard families the isochronicity and monotonicity problems were studied

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