International Journal of Bifurcation and Chaos, Vol. 33, No. 16 (2023) 2350196 (10 pages)

DOI: 10.1142/S0218127423501961

The Extended 16th Hilbert Problem for Discontinuous Piecewise Systems Formed by Linear Centers and Linear Hamiltonian Saddles Separated by a Nonregular Line

Jaume Llibre
Departament de Matemàtiques, Universitat Autònoma de Barcelona, Edifici C, 08193 Bellaterra, Barcelona, Catalonia, Spain
jaume.llibre@uab.cat

Claudia Valls
Departamento de Matemática, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais 1049–001, Lisboa, Portugal
cvalls@math.ist.utl.pt

Received September 13, 2023; Revised October 12, 2023

We study discontinuous piecewise linear differential systems formed by linear centers and/or linear Hamiltonian saddles and separated by a nonregular straight line. There are two classes of limit cycles: the ones that intersect the separation line at two points and the ones that intersect the separation line in four points, named limit cycles of type II_2 and limit cycles of type II_4, respectively. We prove that the maximum numbers of limit cycles of types II_2 and II_4 are two and one, respectively. We show that all these upper bounds are reached providing explicit examples.

Keywords: Piecewise linear differential systems; discontinuity nonregular line; limit cycles.

1. Introduction and Statement of the Main Result

The 16*th Hilbert problem* which consists of finding an upper bound for the maximum number of limit cycles (periodic orbits of a differential system isolated in the set of all periodic orbits of that system) that a given class of differential systems can exhibit, is in general a very hard and unsolved problem. Only for very few classes of differential systems this problem has been solved. We note that limit cycles play an important role for explaining physical phenomena, see for instance, the limit cycle of van der Pol equation [van der Pol, 1920, 1926], or the one of the Belousov–Zhabotinsky model [Belousov, 1959; Zhabotinsky, 1964], etc. and so there has been an intense active research on the limit cycles for many distinct smooth differential systems.

In recent years, the study of the limit cycles has been extended to discontinuous planar piecewise differential systems (that is, differential systems whose vector field is discontinuous on some curve). These limit cycles exhibit more complex dynamical behavior because two kinds of limit cycles can appear in discontinuous piecewise linear differential systems due to the existence of the discontinuous curve. Following Filippov's convention [Filippov, 1988] they are the sliding limit cycles (those that contain some pieces of the discontinuity curve) and