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On the Integrability of a Four-Prototype Rössler System

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Abstract
We consider a four-prototype Rossler system introduced by Otto Rössler among others as prototypes of the simplest autonomous differential equations (in the sense of minimal dimension, minimal number of parameters, minimal number of nonlinear terms) having chaotic behavior. We contribute towards the understanding of its chaotic behavior by studying its integrability from different points of view. We show that it is neither Darboux integrable, nor C^1-integrable.

Keywords First integrals · Invariant algebraic surfaces · Exponential factors · Rössler system

Mathematics Subject Classification 34C05

1 Introduction and Statement of the Main Results

The question whether a differential system admits a first integral (see Sect. 2 for its precise definition) is of fundamental nature, first because the first integrals provide conservation laws for the system that enables to lower its dimension (once restricted to a precise value of it), and second because knowing a sufficient number of them allows to determine its orbits (solving it), at least in an explicit functional form because to compute the intersection of the different constant hypersurfaces defined by the first integrals is in general difficult to do. The distinction between integrable and nonintegrable systems has the qualitative implication of regular motion versus chaotic

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