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Polynomial differential systems with hyperbolic limit cycles

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ABSTRACT

One of the main problems of the qualitative theory of the planar differential equations is the study their limit cycles. In this paper given an algebraic curve of degree n we characterize the planar polynomial differential systems of degree greater or equal than n which admit the ovals of the algebraic curve as hyperbolic limit cycles.

1. Introduction and statement of the main results

The second part of the 16th Hilbert problem aims to obtain the maximum number of limit cycles of the polynomial differential equation

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

where the dot means derivative with respect to the independent variable t and P, Q are polynomials. There is an extensive literature on the existence, number and stability of limit cycles for the differential equation (1) (see for instance [3,5,7,8,14,19] and the references therein). It is a very hard problem to know the existence of limit cycles for a given polynomial differential equation and it is even harder to know its exact analytical expression and this has been done for very few and specific cases.

The aim of this paper is to provide a contribution in this direction by determining the number of limit cycles and their expression for certain polynomial differential systems (1). Guided by [1,6,10,12,13,15,16] we will give polynomial differential systems where we will provide the number and explicit form of the limit cycles by just choosing the components of the system in a clever way.

The limit cycles studied here are algebraic limit cycles, that is the limit cycle is contained in an algebraic curve. Other papers where algebraic limit cycles have been studied are [4,11,18].

Before stating the main result of the paper we introduce some preliminary definitions. Let $\mathbb{R}[x, y]$ be the ring of polynomials with real coefficients. Given $U \in \mathbb{R}[x, y]$ the algebraic curve $U = 0$ is called *invariant* of the polynomial differential equation (1) if for some polynomial $K \in \mathbb{R}[x, y]$ called *the cofactor* of the algebraic curve, we have

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