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# Phase portraits on the unit sphere of the stretch-twist-fold flow

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**Abstract.** The so-called stretch-twist-fold flow consists in a Stokes flow depending on two parameters defined in a unit closed ball $\bar{B}$ that is associated with the motion of a fluid particle coming from the dynamo theory, and it models a mechanism for studying the magnetic field of the Earth and the Sun. Here for the first time, we classify all the local phase portraits of its equilibrium points, and we provide the global phase portraits on the 2-dimensional sphere of the boundary of the ball $\bar{B}$.

**Keywords:** stretch-twist-fold flow, equilibrium points, local phase portraits, global phase portraits.

## 1 Introduction and statement of main results

The stretch-twist-fold flow is a special case of the Stokes flow coming from the dynamo theory. More precisely, it is a two-parameter family of a three-dimensional incompressible flow defined in the unit closed ball that is associated with the fluid particle motion coming from the dynamo theory, and it was devised to represent the stretch-twist-fold action that is believed to be most conductive of the so-called "fast dynamo action" in magnetohydrodynamics; see for more details [11,17,18]. In other words, it is a model for studying the origin, maintenance and amplification of the magnetic field of the celestial bodies such as stars and planets.

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