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Reversible global centres with quintic homogeneous nonlinearities

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ABSTRACT

A *centre* of a differential system in the plane $\mathbb{R}^2$ is a singular point p having a neighbourhood U such that $U \setminus \{p\}$ is filled of periodic orbits. A *global centre* is a centre p such that $\mathbb{R}^2 \setminus \{p\}$ is filled of periodic orbits. To determine if a given differential system has a centre is in general a difficult problem, but it is even harder to know if it has a global centre. In the present paper we deal with the class of polynomial differential systems of the form

$$\dot{x} = -y + P(x,y), \quad \dot{y} = x + Q(x,y), \tag{1}$$

where P and Q are homogeneous polynomials of degree n. It is known that these systems can have global centres only if n is odd and the global centres in the cases $n = 1$ and $n = 3$ are known. Here we work with the case $n = 5$ and we classify the global centres of a four parameter family of systems (1). In particular we illustrate how to study the local phase portraits of the singular points whose linear part is identically zero using only vertical blow ups.

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1. Introduction and statement of the main result

The notion of centre can be found for the first time in the work of Huygens in 1656 on the pendulum clock, see [13,17] but it was not until the works of Poincaré [18] in 1881 and Dulac [6] in 1908 that the rigorous notion of centre appeared in the literature.

A *polynomial differential system* in the plane $\mathbb{R}^2$ (of *degree n*) is a differential system

$$\dot{x} = p(x,y), \quad \dot{y} = q(x,y), \tag{2}$$

where p and q are polynomials in the variables x and y with real coefficients (such that n is the maximum of the degrees of the polynomials p and q). Here the dot denotes derivative with respect to the time t. A polynomial differential system (2) has *degree n* if n is the maximum of the degrees of the polynomials p and q.

If a polynomial differential system has a centre at the origin, then applying a linear change of variables and a rescaling of the time variable, it can be written in one of the

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