



Invariants of polynomial vector fields

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Abstract

We characterize the existence of first integrals and invariants (first integrals depending on the time) for the polynomial vector fields which are invariant under an involution.

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1. Introduction and statement of the main results

Let X be a polynomial vector field in \mathbb{R}^m of degree $d + 1$. Given a polynomial $F(x, y)$, the algebraic hypersurface $F(x, y) = 0$ is an *invariant algebraic hypersurface* of X if there exists a polynomial $K(q, p)$ called *the cofactor* such that $X F = K F$. In this paper we will assume that

$$K(x, y) = \alpha + \bar{K}(x, y),$$

being $\alpha \in \mathbb{R}$ and $\bar{K}(x, y)$ a polynomial without constant terms. It can be proved that K has degree at most d .

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