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On the “traveling pulses” of the limit of the FitzHugh–Nagumo equation when $\varepsilon \to 0$

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A R T I C L E I N F O

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A B S T R A C T

A solution $(u(s), v(s))$ of the differential system

$$u' = v, \quad v' = -cv - u(u-a)(1-u) + w, \quad w' = -(\varepsilon/c)(u - \gamma w).$$

with $a, c, \varepsilon \in \mathbb{R}$ such that $(u(s), v(s)) \to (0, 0)$ when $s \to \pm\infty$ is a traveling pulse of the FitzHugh–Nagumo equation. The limit of this differential system when $\varepsilon \to 0$ gives rise to the polynomial differential system

$$u' = v, \quad v' = -cv - u(u-a)(1-u) + w,$$

where now $a, c, w \in \mathbb{R}$. We give the complete description of its phase portraits in the Poincaré disc (i.e. in the compactification of $\mathbb{R}^2$ adding the circle $\mathbb{S}^1$ of the infinity) modulo topological equivalence.

1. Introduction and statement of the main results

Numerous problems of applied mathematics, or in physics, chemistry, economics, . . . are modeled by polynomial differential systems. Excluding linear differential systems the quadratic polynomial differential systems are the ones with the lowest degree of complexity, and the large bibliography on them proves their relevance, see the books [1–3] and the surveys [4,5]. After the quadratic polynomial differential systems come the cubic ones, which also have many applications.

This paper deals with the traveling waves of the FitzHugh–Nagumo equations

$$u_t = u_{xx} + f(u) - w, \quad w_t = \varepsilon(u - \gamma w), \tag{1}$$

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