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Limit cycles of piecewise differential systems with linear Hamiltonian saddles and linear centres

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ABSTRACT

We study the continuous and discontinuous planar piecewise differential systems formed by linear centres together with linear Hamiltonian saddles separated by one or two parallel straight lines. When these piecewise differential systems are either continuous or discontinuous separated by one straight line, they have no limit cycles. When these piecewise differential systems are continuous and are separated by two parallel straight lines they do not have limit cycles. On the other hand, when these piecewise differential systems are discontinuous and separated by two parallel straight lines (either two centres and one saddle, or two saddles and one centre), we show that they can have at most one limit cycle, and that there exist such systems with one limit cycle. If the piecewise differential systems separated by two parallel straight lines have three linear centres or three linear Hamiltonian saddles it is known that they have at most one limit cycle.

ARTICLE HISTORY

Received 7 July 2021 Accepted 31 January 2022

KEYWORDS

Limit cycles; linear centres; linear Hamiltonian saddles; continuous piecewise linear differential systems; discontinuous piecewise differential systems; first integrals

2000 MATHEMATICS SUBJECT CLASSIFICATIONS

34C05; 34C23; 34C25; 34C29

1. Introduction and statement of the main results

A limit cycle is a periodic orbit of a differential system isolated in the set of all periodic orbits of that system. The study of the limit cycles goes back essentially to Poincaré [24] at the end of the nineteenth century.

The existence of limit cycles became important in the applications to the real world, because many phenomena are related with their existence, see for instance the Van der Pol oscillator [27,28], or the Belousov–Zhabotinskii reaction which is a classical reaction of non-equilibrium thermodynamics appearing in a non- linear chemical oscillator [3,29]. The study of the continuous piecewise linear differential systems separated by one or two parallel straight lines appears in a natural way in the control theory, see for instance the books [2,10,12,13,18,23]. The easiest continuous piecewise linear differential systems are formed by two linear differential systems separated by a straight line. It is known that such systems have at most one limit cycle, see [8,15,20,21].

The study of the discontinuous piecewise linear differential systems separated by straight lines goes back to Andronov et al. [1] and until nowadays they have special

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