Physics of the Dark Universe 35 (2022) 100954

Contents lists available at ScienceDirect

Physics of the Dark Universe

journal homepage: www.elsevier.com/locate/dark

Global dynamics for the Szekeres system with non-zero cosmological constant

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ARTICLE INFO

Article history:
Received 11 December 2021
Accepted 10 January 2022

MSC:
34C29
34C25
47H11

Keywords:
Szekeres system
Positive cosmological constant
Global dynamics
Poincaré sphere

ABSTRACT

The Szekeres system with cosmological constant term describes the evolution of the kinematic quantities for the Einstein field equations in dimension four. It is a Hamiltonian system (with Hamiltonian H). We restrict the dynamics on each one of the level surfaces $H = h$ with $h \in \mathbb{R}$ and using the Poincaré compactification on $\mathbb{R}^3$ we analyze the global dynamics of the Szekeres system.

It was known that the Szekeres system with cosmological constant term exhibits an attractor in the finite regime. Here we provide a new proof of the finite attractor and additionally we prove that also exhibits a repulsor in the finite regime, and that at infinity there is an attractor and a repulsor.

1. Introduction and statement of the main results

A Szekeres system represents the diagonal of the Einstein field equations $G - \Lambda g = T$ for a gravitational model where the energy–momentum tensor T is that of a pressureless inhomogeneous fluid, $\Lambda > 0$ is the cosmological constant and $G = R - \frac{1}{2}Rg$ is the Einstein tensor for the background space, see for details [1]. The Szekeres system also can be obtained from the model of the silent Universe system, see for details [2]. Because of its importance in physical applications it has been widely investigated in the literature using Darboux functions, Jacobi's multiplier method, Painlevé method, Lie symmetries, ...

It was proved in [3] (see also [4,5]) that with appropriate variables, the Szekeres system admits the Hamiltonian formalism with Hamiltonian

$$H = p_x p_y - \frac{\Lambda}{3}xy + \frac{x}{y^2},$$

and then its equations of motion are

$$\begin{aligned}
\dot{x} &= p_y,\\
\dot{y} &= p_x,\\
\dot{p_x} &= \frac{\Lambda}{3}x + \frac{2x}{y^3},\\
\dot{p_y} &= \frac{\Lambda}{3}y - \frac{1}{y^2},
\end{aligned}$$

where the dot denotes derivative with respect to the time t.

We introduce a rescaling of the time by $dt = y^3\, ds$ and with this new time the differential system becomes

$$\begin{aligned}
\dot{x} &= p_y y^3,\\
\dot{y} &= p_x y^3,\\
\dot{p_x} &= \frac{\Lambda}{3}xy^3 + 2x,\\
\dot{p_y} &= \frac{\Lambda}{3}y^4 - y,
\end{aligned}$$

now the dot denotes derivative with respect the new independent variable s.

We shall restrict the study of the dynamics of this system to each energy level $H = h$, $h \in \mathbb{R}$. In particular setting $H = h$ and solving this equation with respect to the variable p_x we get

$$p_x = \frac{-3x + 3hy^2 + xy^3\Lambda}{3p_y y^2},$$

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https://doi.org/10.1016/j.dark.2022.100954
2212-6864/