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The Markus–Yamabe Conjecture for Discontinuous Piecewise Linear Differential Systems in $\mathbb{R}^n$ Separated by a Conic $\times\, \mathbb{R}^{n-2}$

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Abstract

In 1960 Markus and Yamabe made the conjecture that if a C^1 differential system $\dot{x} = F(x)$ in $\mathbb{R}^n$ has a unique equilibrium point and $DF(x)$ is Hurwitz for all $x \in \mathbb{R}^n$, then the equilibrium point is a global attractor. This conjecture was completely solved in 1997 and it turned out to be true in $\mathbb{R}^2$ and false in $\mathbb{R}^n$ for all $n \geq 3$. In (The Markus–Yamabe conjecture for continuous and discontinuous piecewise linear differential systems, 2020) the authors extended the Markus–Yamabe conjecture to continuous and discontinuous piecewise linear differential systems in $\mathbb{R}^n$ separated by a hyperplane, they proved for the continuous systems that the extended conjecture is true in $\mathbb{R}^2$ and false in $\mathbb{R}^n$ for all $n \geq 3$, but for discontinuous systems they proved that the conjecture is false in $\mathbb{R}^n$ for all $n \geq 2$. In this paper first we show that there are no continuous piecewise linear differential systems separated by a conic$\times\mathbb{R}^{n-2}$ except the linear differential systems in $\mathbb{R}^n$. And after we prove that the extended Markus–Yamabe conjecture to discontinuous piecewise linear differential systems in $\mathbb{R}^n$ separated by a conic$\times\mathbb{R}^{n-2}$ is false in $\mathbb{R}^n$ for all $n \geq 2$.

Keywords Markus–Yamabe conjecture · Hurwitz matrix · Discontinuous piecewise linear differential systems

Mathematics Subject Classification 34C05 · 34C07 · 34C08

1 Introduction and Statement of the Results

Consider a C^1 differential system $\dot{x} = F(x)$ defined in $\mathbb{R}^n$ and having an equilibrium point at the origin of coordinates. If $DF(0)$ is Hurwitz (i.e. all eigenvalues of $DF(0)$ have negative real

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