Journal of Mathematical Physics | ARTICLE | scitation.org/journal/jmp

Dynamics of the Szekeres system

Cite as: J. Math. Phys. 62, 082502 (2021); doi: 10.1063/5.0054051
Submitted: 14 April 2021 • Accepted: 27 July 2021 •
Published Online: 9 August 2021

Jaume Llibre[1] and Claudia Valls[2,a)]

AFFILIATIONS

[1] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain
[2] Departamento de Matemática, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

[a)] **Author to whom correspondence should be addressed:** cvalls@math.tecnico.ulisboa.pt

ABSTRACT

The Szekeres model is a differential system in $\mathbb{R}^4$ that provides the solutions of the Einstein field equations in the presence of irrotational dust. This differential system is integrable with two rational first integrals and one analytic first integral. We characterize the qualitative behavior of all the orbits of the Szekeres system in the function of the values of the two rational first integrals.

I. INTRODUCTION AND STATEMENT OF THE RESULTS

The Szekeres model introduced in Ref. 1 is a differential system in $\mathbb{R}^4$ that provides the solutions of the Einstein field equations in the presence of irrotational dust. In Ref. 2, it is shown that a limit case of the Szekeres system is the Lemaître–Tolman system. The Szekeres system is

$$\begin{aligned}\dot{\rho} &= -\theta\rho,\\ \dot{\theta} &= -\frac{1}{3}\theta^2 - 6\sigma^2 - \frac{1}{2}\rho,\\ \dot{\sigma} &= \sigma^2 - \frac{2}{3}\theta\sigma - E,\\ \dot{E} &= -3E\sigma - \theta E - \frac{1}{2}\rho\sigma,\end{aligned} \tag{1}$$

with ρ being the energy density, θ being the expansion scalar, σ being the shear, and E being the Weyl tensor. As usual, the dot denotes the derivative with respect to the time t.

In Ref. 3, it was proved that if in the silent Universe system we take $\sigma_1 = \sigma_2 = \sigma$ and $E_1 = E_2 = E$, then we get the Szekeres system (the eigenvalues of the shear tensor are σ_1 and σ_2 and the traceless components of the Weyl tensor are E_1 and E_2). Some authors have used the Szekeres system for studying the propagation of light in nonhomogeneous universe models or for describing the formation and evolution of the Universe (see Refs. 4–8 to cite just a few). These works clearly indicate the importance of studying the motion of the Szekeres system near infinity.

In Ref. 9, it was proved that the Szekeres system is integrable with two rational first integrals and a third analytic one, the three of them being independent. The rational ones are

$$\begin{aligned}F &= \frac{\left(-18E - 3\rho + (\theta + 3\sigma)^2\right)^3}{(6E + \rho)^2},\\ H &= \frac{\left(3\theta\sigma(2E + \rho) - E\left(18E + 2\theta^2 + 3\rho\right) + 9\sigma^2(4E + \rho)\right)^3}{\rho^3(\rho + 6E)^2}.\end{aligned}$$