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**Results in Mathematics**



# Configurations of the Topological Indices of the Planar Polynomial Differential Systems of Degree $(2, m)$

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**Abstract.** Using the Euler–Jacobi formula there is a relation between the singular points of a polynomial vector field and their topological indices. Using this formula we obtain the configuration of the singular points together with their topological indices for the polynomial differential systems $\dot{x} = P(x, y)$, $\dot{y} = Q(x, y)$ with degree of $P$ equal to 2 and degree of $Q$ equal to $m$ when these systems have $2m$ finite singular points.

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**Keywords.** Euler–Jacobi formula, singular points, topological index, polynomial differential systems, Berlinskii's theorem.

## 1. Introduction and Statement of the Main Results

Consider in $\mathbb{R}^2$ the polynomial differential system

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

where $P(x, y)$ and $Q(x, y)$ are real polynomials of degrees 2 and $m$, respectively, or simply of degree $(2, m)$.

The motivation of our paper comes from the fact that for the planar quadratic polynomial differential systems (i.e. the ones of degree $(2, 2)$) the characterization of all configurations of the indices of the singular points of all systems that have four singular points is the well-known Berlinskii's Theorem proved in [2, 5] and reproved in [4] using the Euler–Jacobi formula. More precisely, the Berlinskii's Theorem can be stated as follows: *Assume that a real*

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