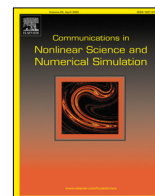


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Research paper

On the periodic orbits of the continuous–discontinuous piecewise differential systems with three pieces separated by two parallel straight lines

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ABSTRACT

During these last twenty years many papers have been published about the piecewise differential systems in the plane. The increasing interest on these kind of differential systems mainly is due to their big number of applications for modeling many natural phenomena. As usual one of the main difficulties for understanding the dynamics of the differential systems in the plane consists in controlling their periodic orbits and mainly their limit cycles. Therefore there are a big number of papers studying the existence or non-existence of periodic orbits of the continuous and discontinuous piecewise differential systems, but as far as we know this paper will be one of the first papers studying the periodic orbits of a class of piecewise differential systems such that in a part of the line of separation between the two differential systems forming the piecewise differential system is continuous and in the other part it is discontinuous.

Thus we shall study the piecewise differential systems separated by two parallel straight lines, in one of these straight lines the piecewise differential system is continuous and in the other discontinuous, moreover in the three pieces of these piecewise differential systems we put arbitrary Hamiltonian systems of degree one. We obtain that such kind of continuous–discontinuous piecewise differential systems cannot have limit cycles, but they can have a continuum of periodic orbits.

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1. Introduction and results

Around 1920's started the first studies on the piecewise differential systems, see the book [1] of Andronov, Vitt and Khaikin. Nowadays the piecewise differential systems continue receiving the attention of many researchers because they model many phenomena appearing in mechanics, electronics, economy, etc., see the books of di Bernardo et al. [2] and Simpson [3], and also the survey of Makarenkov and Lamb [4], and the hundreds of references mentioned in these last references.

The simplest continuous piecewise differential systems are the ones formed by two pieces separated by one straight line in the plane \mathbb{R}^2 and having in each piece a linear differential systems. In 1991 Lum and Chua [5,6] conjectured that such piecewise differential systems have at most one limit cycle. We recall that a limit cycle is an isolated periodic orbit

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