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PERIODIC POINTS FOR TRANSVERSAL MAPS ON SURFACES

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ABSTRACT. In this article we investigate the set of least periods of transversal maps $f : S \to S$ on surfaces using the growth rate of the sequence $L(f^k)$ of Lefschetz numbers. We present several criteria, in terms of the homology homomorphisms induced by f, which ensure that the map f has periodic orbits corresponding to a cofinite set of odd periods. We also provide sufficient conditions for the existence of a cofinite set of power of two periods corresponding to periodic orbits.

1. Introduction.

Compact connected 2-dimensional manifolds will be called *surfaces.* An orientable surface without boundary is homeomorphic to the sphere $\mathbb{S}^2$, to the torus $\mathbb{T}^2$, or to the connected sum of n tori with $n \geq 2$; i.e., to the n-holed torus. The *genus* g of an orientable surface without boundary is the number of torus summands. An orientable surface with boundary is homeomorphic to an orientable surface without boundary minus a finite number b of open discs. We shall also be interested in compact *non-orientable* surfaces with boundary and finite genus. For $g \geq 0$, a *non-orientable surface of genus g with b boundary components* is the connected sum of $g+1$ copies of the projective plane $\mathbb{P}^2$ with b open discs removed.

A *fixed point* of f is a point p of the surface S such that $f(p) = p$. Denote the totality of fixed points by $\text{Fix}(f)$. The point $p \in S$ is *periodic with period m* if $p \in \text{Fix}(f^m)$ but $p \notin \text{Fix}(f^k)$ for all $0 \leq k < m$. Let $\text{Per}(f)$ denote the set of all periods of periodic points of f.

Suppose that S is an orientable surface of genus g having b boundary components. We put $b = 0$ if S is without boundary. A continuous map $f : S \to S$ induces homomorphisms $f_{*k} : H_k(S;\mathbb{Q}) \to H_k(S;\mathbb{Q})$ for $k =$