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Entropy and periods for continuous graph maps

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Abstract
For continuous self-maps on topological graphs, we provide new relationships between their topological entropy, their homology and their periods.

Keywords Topological entropy · Periods · Lefschetz numbers · Lefschetz zeta function · Graphs

Mathematics Subject Classification 37C25 · 37C30 · 37E15 · 55M20

1 Introduction

In the field of dynamical systems, an important question is: what kind of topological conditions implies positive topological entropy? In the present article, we deal with this question for continuous self-maps on graphs, not homotopic to a point or a circle. We explore some relationships between the topological entropy of a graph map, the induced map on homology and its periodic structure. In particular, we give sufficient conditions on a graph map (in terms of its Lefschetz numbers, Lefschetz zeta function and/or the characteristic polynomial of the induced map on homology) to have positive topological entropy (Theorem 3). All these concepts are defined in Sect. 2. In Theorem 4, we show that these conditions are not necessary conditions, in particular, we prove that given any product of cyclotomic polynomial of total degree s, there exists a graph map on G_s (a bouquet of s circles), so that the characteristic polynomial of the induced map on homology is the given polynomial. Theorems 7 and 8 and Corollaries 9 and 10 show how the periodic structure of a graph map on G_s is determined only by first s Lefschetz numbers of the map.

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