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Complete integrability of vector fields in $\mathbb{R}^N$

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ABSTRACT

We give necessary and sufficient conditions for the complete integrability of first order N–dimensional differential systems.
We propose a new method to determine in the Jacobi Theorem the last $N-1$ first integral for the complete integrability of an N–dimensional differential system with $N-2$ independent first integrals and with a Jacobi multiplier.
As an application we study the complete integrability of some 3–dimensional differential systems, more precisely the complete integrability of the asymmetric and symmetric May–Leonard differential systems.

1. Introduction

For the N–dimensional nonlinear differential systems the existence of $K < N-1$ independent first integrals means that these systems are *partially integrable*. The existence of $N-1$ independent first integrals means that the system is *completely integrable*, i.e. for such systems the intersection of the $N-1$ hypersurfaces obtained fixing the $N-1$ first integrals provide the trajectories of the differential system.

We give necessary and sufficient conditions under which the differential system

$$\dot{x}_j = X_j(x_1, \ldots, x_N), \quad \text{for} \quad j = 1, \ldots, N, \tag{1}$$

or its associated vector field

$$\mathcal{X} = X_1 \frac{\partial}{\partial x_1} + X_2 \frac{\partial}{\partial x_2} + \ldots + X_N \frac{\partial}{\partial x_N},$$

is completely integrable. Here $X_j : U \longrightarrow \mathbb{R}^N$ are C^1 functions defined in an open subset $U \subseteq \mathbb{R}^N$. Using these necessary and sufficient conditions we propose a new method to determine the last $N-1$ first integral in the Jacobi Theorem for the complete integrability of the differential system (1) having $N-2$ independent first integrals and a Jacobi multiplier.

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