



The center problem for the class of $\mathcal{A} - \mathcal{Q}$ differential systems

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Abstract

The center problem, i.e. distinguish between a focus and a center, is a classical problem in the qualitative theory of planar differential equations which go back to Darboux, Poincaré and Liapunov. Here we solve the center problem for the class of planar analytic or polynomial differential systems

$$\dot{x} = -y + X = -y + \sum_{j=2}^k X_j, \quad \dot{y} = x + Y = x + \sum_{j=2}^k Y_j, \quad k \leq \infty,$$

where $X_j = X_j(x, y)$ and $Y_j = Y_j(x, y)$ are homogenous polynomials of degree $j > 1$, under the condition

$$(x^2 + y^2) \left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} \right) = \mu(xX + yY) \quad \text{with} \quad \mu \in \mathbb{R} \setminus \{0\}.$$

Moreover we prove that these centers are weak centers, and additionally we provide their first integrals.

Keywords First integral · Poincaré–Liapunov first integral · Analytic planar differential system · Polynomial differential system · Weak center

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