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A GRAVITATIONAL APPROACH TO THE TITIUS-BODE LAW

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ABSTRACT

The Titius-Bode law tells us that the distances of the planets sufficiently far from the Sun roughly follow a geometric progression of ratio equal to 2. We give a gravitational explanation of this fact taking into account the motion of the solar system around the center of mass of the galaxy.

I. INTRODUCTION

Let a be the mean distance from the Sun to a planet. If the planetary orbit is an ellipse with the Sun at one focus (Kepler's first law), then a is the semimajor axis of the ellipse.

The Titius-Bode law says that the distances a_k from the Sun to the planets in astronomical units are approximately given by the formula $a_k = 0.4 + 0.3 \times 2^k$ for $k = -\infty, 0, 1, 2 ..., 7$ (except for Neptune, see Table I or Voight (1974, pgs. 32–79)). This law was observed by Johann Daniel Titius in 1766 and divulged by Johann Elert Bode in 1772. Of course, at that epoch, Uranus, the asteroids, Neptune, and Pluto were unknown.

Let n_k be the motion of the k th planet with respect to the $(k-1)$th planet when the mean motion of this last is chosen equal to unity. Then, if the orbits of the planets are Keplerian (by Kepler's third law), we have $n_k = (a_k/a_{k-1})^{-3/2}$.

The Titius-Bode law tells us that the distances of the planets sufficiently far from the Sun roughly follow a geometric progression of ratio equal to 2. This fact will be called the Titius-Bode limit law. Notice that if $a_k/a_{k-1} \approx 2$, then $n_k \approx 1/3$ (in fact, if $a_k/a_{k-1} = 3^{2/3} = 2.08$, then $n_k = 1/3$). Therefore, another version of the Titius-Bode limit law says that the period of the k th planet must be close to three times the period of the $(k-1)$th planet.

Our goal is to give a gravitational explanation of the existence of this Titius-Bode limit law in the solar system, taking into account the motion of the solar system around the center of the galaxy.

In fact, we shall consider a simplified model of a solar system formed by two planets, the Sun, and center of the galaxy. In this model we shall prove that if the inner planet describes a circular orbit, then the best nearly circular linearly stable periodic orbit candidate to be the orbit of the outer planet is such that its mean distance from the Sun is close to twice the inner planet's mean distance from the Sun. This will be our gravitational approach to the Titius-Bode limit law.

We assume the following peculiarities of the planetary orbits of the solar system which do not follow from the law of gravitation: (a) all the planets approximately in a plane, (b) planetary orbits nearly circular, rotations all in the same sense (direct). Of course, these two properties seem to be due to the cosmogony (origin) of the solar system (see Voight 1974, p. 104).

Different papers have been published on the Titius-Bode law (see the references given in the paper of Horedt, Pop, and Ruck 1977). In this paper some numerical examples concerning the Titius-Bode law are presented. On the other hand, Ovenden (1973) applied the principle of least interaction to the solar system in order to discuss the possible physical reasons for the distribution of planetary orbits. However, none of these works takes into account the motion of the solar system around the center of mass of the galaxy in explaining the Titius-Bode law.

II. THE MODEL

We consider a restricted four-body problem. The four bodies are the Sun, an inner planet, and the center of mass of the galaxy as the primaries, and the fourth body a massless outer planet, attracted by the previous three primaries but not influencing their motion.

In a restricted problem of three bodies, the motion of the two primaries satisfies precisely the equations of motion of the two-body problem. Consequently, the logical generalization is to establish a solution of the problem of three bodies and find the motion of the fourth body under the gravitational attraction of the three primaries. Since the exact solution is not known for the three-body problem given by the center of mass of the galaxy, the Sun, and the inner planet, we assume the motions of the three primaries and accept an ap-

TABLE I. Data for the planets of the solar system.

	k	a_k	a observed	Period of revolution	mass
Mercury	$-\infty$	0.4	0.39	88ᵈ00	0.06
Venus	0	0.7	0.72	224ᵈ70	0.81
Earth	1	1.0	1.00	1ʸ00	1.00
Mars	2	1.6	1.52	1ʸ86	0.11
asteroids	3	2.8	2.90		
Jupiter	4	5.2	5.20	11ʸ86	317.8
Saturn	5	10.0	9.55	29ʸ46	95.1
Uranus	6	19.6	19.20	84ʸ02	14.5
Neptune			30.09	164ʸ80	17.2
Pluto	7	38.8	39.50	247ʸ70	0.1