ON THE FULL PERIODICITY KERNEL FOR σ MAPS

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A connected finite regular graph (or just a *graph* for short) is a pair consisting of a connected Hausdorff space G and a finite subspace V (points of V are called *vertices*) such that the following conditions hold:

(1) $G \setminus V$ is the disjoint union of a finite number of open subsets $e_1, \dots, e_k$ called *edges*. Each e_i is homeomorphic to an open interval of the real line.

(2) The boundary, $\text{cl}(e_i) \setminus e_i$, of the edge e_i consists of two distinct vertices, and the pair $(\text{cl}(e_i), e_i)$ is homeomorphic to the pair $([0,1], (0,1))$.

A vertex v which belongs to the boundary of at least three different edges is called a *branching point* of G.

A *G map* f is a continuous self-map of G having all branching points of G as fixed points.

A point x of G will be called *periodic* with respect to f of *period* n if n is the smallest positive integer such that $f^n(x) = x$. The set $\{x, f(x), \dots, f^{n-1}(x)\}$ is called the *periodic orbit* of x. We denote by $\text{Per}(f)$ the set of periods of all periodic points of f, and by $\mathbf{N}$ the set of positive integers.

A G map f has *full periodicity* if $\text{Per}(f) = \mathbf{N}$. The set $K \subseteq \mathbf{N}$ is a *full periodicity kernel of G* if it satisfies the following two conditions:

(1) If f is a G map and $K \subseteq \text{Per}(f)$, then $\text{Per}(f) = \mathbf{N}$.

(2) If $S \subseteq \mathbf{N}$ is a set such that for every G map f, $S \subseteq \text{Per}(f)$ implies $\text{Per}(f) = \mathbf{N}$, then $K \subseteq S$.

Notice that for a given G if there is a full periodicity kernel, then it is unique.

The full periodicity kernel has been computed for the closed interval, the circle and the Y, more precesily:

(I) Let I be the closed interval $[0,1]$. Then the set $\{3\}$ is the full periodicity kernel of I (see [**10**] and [**8**]).

(S) Let S^1 be the circle. Then the set $\{1,2,3\}$ is the full periodicity kernel of S^1 (see [**4**] and [**7**]).

(Y) Set $Y = \{z \in \mathbf{C} : z^3 \in [0,1]\}$. The set $\{2,3,4,5,7\}$ is the full periodicity kernel of Y (see [**9**] and [**1**]).

In this paper we characterize the full periodicity kernel of σ, where σ is the topological space formed by the points (x,y) of $\mathbf{R}^2$ such that either $x^2 + y^2 = 1$, or $0 \le x \le 2$ and $y = 1$. Then our main result is the following.