



# Qualitative study of the Selkov model

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## Abstract

The Selkov oscillator was formulated in 1968 and now it is a classical model for studying the glycolysis. It is a differential system of two equations depending on two parameters in dimensionless variables. When the two equations are polynomials we prove that the Selkov system is not Liouvillian integrable. Additionally, we prove that the polynomial Selkov system for any integer  $n \geq 1$  has nine distinct phase portraits in the Poincaré disk.

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## 1. Introduction and statement of the main results

The living organisms obtain energy from sugar using a process called glycolysis. Experimental observations detected that when the input rate of sugar is constant then the subproducts of the glycolysis oscillate on time. Based on these observations Higgins [12] in 1964 provided a mathematical model in order to understand better this phenomenon. Higgins' model was improved in 1968 by Selkov [19]. Thus the Selkov model is given by the differential system of two equations

$$\begin{aligned}\dot{x} &= 1 - xy^\nu, \\ \dot{y} &= ay(-1 + xy^{\nu-1}),\end{aligned}$$

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