Integrability and dynamics of a simplified class B laser system

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J. Llibre[1,a] and C. Pantazi[2,b]

AFFILIATIONS

[1] Dept. Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain
[2] Departament de Matemàtiques, Universitat Politècnica de Catalunya, (EPSEB), Av. Doctor Marañón, 44–50, 08028 Barcelona, Spain

[a] jllibre@mat.uab.cat
[b] **Author to whom correspondence should be addressed:** chara.pantazi@upc.edu

ABSTRACT

A simplified class B laser system is a family of differential polynomial systems of degree two depending on the parameters a and b. Its rich dynamics has already been observed in 1980s, see Arecchi *et al.* [Opt. Commun. **51**, 308–314 (1984)] and Politi *et al.* [Phys. Rev. A **33**, 4055 (1986)], and still nowadays, it attracts the interest of the researchers. In this paper, we characterize its dynamics near infinity for all values of the parameters. When $a = 0$, the partial integrability was already proved by Oppo and Politi [Z. Phys. B Con. Mat. **59**, 111–115 (1985)]. Here, we prove that for $a = 0$, it is completely integrable with two independent first integrals given by Liouvillian functions, and we present a complete study of its dynamics. When $a \neq 0$, we study its dynamics in the Poincaré ball $\mathbb{B}^3$, i.e., the interior of this ball is identified with $\mathbb{R}^3$ and its boundary the two-dimensional sphere $\mathbb{S}^2$ is identified with the infinity of $\mathbb{R}^3$.

A laser is a device that emits light after stimulation of atoms or molecules. It is a key component of many everyday products with further uses in manufacturing and in medical procedures. In particular, a simplified class B laser system is a two parametric family of nonlinear differential equations in three dimensions that since 1980s attracts the interest of the researchers. In this work, we characterize completely its dynamics near the infinity for all values of the parameters using the Poincaré ball $\mathbb{B}^3$. Our results, purely analytic, are in accordance with some numerical results already appeared in the literature. When a parameter is zero, we succeed to prove the complete integrability of this system, and hence, we improve some previous known results. We note that for $a = 0$, the partial integrability is already proved in Ref. 1.

I. INTRODUCTION AND STATEMENT OF THE MAIN RESULT

The word LASER is an acronym for Light Amplification by Stimulated Emission of Radiation. It is a device that stimulates atoms or molecules to emit light at particular wavelengths and amplifies that light, producing a very narrow beam of radiation. Lasers are used in optical disk drives, laser printers, barcode scanners, DNA sequencing instruments, fiber-optic, and free-space optical communication, semiconducting chip manufacturing, laser surgery and skin treatments, cutting and welding materials, and in laser lighting displays.

Lasers are typical models that can be described using nonlinear dynamics. In 1984, the authors of Ref. 2 classify the lasers into following three classes according to the damping rates $\gamma_\perp, \gamma_\parallel$, and k (rates of polarization, population, and field, respectively); see for more details, Ref. 3.

Class A: $\gamma_\perp \simeq \gamma_\parallel \gg k$. Polarization and population decay much faster than the field, and the dynamics are governed by a single field equation.

Class B: $\gamma_\perp \gg k \gtrsim \gamma_\parallel$. The population decays slowly, so that the dynamics are described by two coupled rate equations. By injection of an external signal into a homogeneous line, the dynamics can be described by a system of three nonlinear coupled equations.

Class C: $\gamma_\parallel \simeq \gamma_\perp \simeq k$. The three decay rates for polarization, population, and field are of the same order of magnitude.

In particular, class B lasers become reliable devices for studying chaos and generalized multistability; see Ref. 4.