



Algebraic and topological classification of homogeneous quartic vector fields in the plane

Jaume Llibre¹ · Y. Paulina Martínez² · Claudio Vidal²

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Abstract

We provide canonical forms for the homogeneous polynomials of degree five. Then we characterize all the phase portraits in the Poincaré disk for all quartic homogeneous polynomial differential systems. More precisely, there are exactly 23 different topological phase portraits for the quartic homogeneous polynomial differential systems.

Keywords Quartic homogeneous polynomial differential systems · Homogeneous polynomial vector fields · Phase portraits

Mathematics Subject Classification Primary 37C10 · Secondary 34C05

1 Introduction

We consider a family of polynomial vector fields in the plane of the form

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \quad (1)$$

where P and Q are homogeneous polynomials of degree four (shortly, they will be called *quartic systems*). This work is divided in two parts. First we are going to give all the possible canonical forms for the homogeneous polynomials of degree five, and secondly, we will characterize all the phase portraits in the Poincaré disk of all homogeneous quartic polynomial differential systems (1). For a definition of the Poincaré disk and the local charts we are going to work, see for instance Chapter 5 of [7].

✉ Y. Paulina Martínez
ymartinez@ubiobio.cl

Jaume Llibre
jllibre@mat.uab.cat

Claudio Vidal
clvidal@ubiobio.cl

¹ Departament de Matemàtiques, Facultat de Ciències, Universitat Autònoma de Barcelona, Bellaterra, 08193 Barcelona, Catalonia, Spain

² Departamento de Matemática, Facultad de Ciencias, Universidad del Bío-Bío, Casilla 5–C, VIII–Región, Concepción, Chile