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On the C^1 non-integrability of the autonomous differential systems

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ABSTRACT

In the study of the dynamics of the autonomous differential systems to know the existence or non-existence of first integrals is a relevant fact. These last decades the meromorphic non-integrability of the autonomous differential systems have been studied intensively using the Ziglin's and the Morales–Ramis' theories. Here we study the C^1 non-integrability of the autonomous differential systems, these studies goes back to Poincaré.

It is known that the semiclassical Jayne–Cummings differential system of dimension five has only two independent meromorphic first integrals, namely H and F, and of course any meromorphic function in the variables H and F. Here we illustrate how to study the C^1 non-integrability of the autonomous differential systems showing that the semiclassical Jayne–Cummings differential system of dimension five has only two independent C^1 first integrals H and F, and of course any C^1 function in the variables H and F.

1. Introduction

Ziglin's theory on the meromorphic non-integrability was inspired in the studies of the integrability of the rigid body done by Kovalevskaya. This theory studies the non-integrability of an autonomous differential system using the monodromy group of the variational equation associated to some non-equilibrium solution of the analyzed differential system. Ziglin's theory was improved by the Morales–Ramis theory that considers the Galois differential group instead of the monodromy group of the variational equation, and in general the Galois group is easier to study, see [1]. But both theories only allow to study the non-existence of meromorphic first integrals.

As Arnold said in [2] the mentioned both theories of non-integrability, in a beginning inspired in Kovalevskaya's ideas, go back to Poincaré, because Poincaré for studying the non-integrability of the autonomous differential systems already used the multipliers of the monodromy group of the variational equations associated to periodic orbits. Apparently the mathematical community forgot the results of

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