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LIMIT CYCLES OF CONTINUOUS PIECEWISE DIFFERENTIAL SYSTEMS SEPARATED BY A PARABOLA AND FORMED BY A LINEAR CENTER AND A QUADRATIC CENTER

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This paper is dedicated to my good friend Professor Jibin Li for his 80th birthday

Abstract. Due to their applications to many physical phenomena during these last decades the interest for studying the continuous or discontinuous piecewise differential systems has increased strongly. The limit cycles play a main role in the study of any planar differential system. Up to now the major part of papers which study the limit cycles of the planar piecewise differential systems have considered systems formed by two pieces separated by one straight line.

Here we consider planar continuous piecewise differential systems separated by a parabola.

We prove that the planar continuous piecewise differential systems separated by a parabola and formed by a linear center and a quadratic center have at most one limit cycle. Moreover there are systems in this class exhibiting one limit cycle. So in particular we have solved the extension of the 16th Hilbert problem to this class of differential systems.

1. **Introduction and results.** Andronov, Vitt and Khaikin [1] started in a serious way the study of the piecewise differential systems mainly motivated for their applications to some mechanical systems, and now these systems still continue to receive the attention of many researchers. Recently these differential systems are widely used to model processes appearing in mechanics, electronics, economy, etc., see for instance the books [3] and [29], and the survey [27], as well as the hundreds of references cited there.

One of the main objects in the study of the dynamics of the planar differential systems and in particular in the planar piecewise differential systems are their limit cycles. Recall that a *limit cycle* of a differential system is an isolated periodic orbit in the set of all periodic orbits of the differential system The study of the limit cycles in general is a difficult problem.

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