J. Math. Anal. Appl. 531 (2024) 127895

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

journal homepage: www.elsevier.com/locate/jmaa

Regular Articles

# On the accumulation points of non-periodic orbits of a difference equation of fourth order

A. Linero Bas [a,*], V. Mañosa [b], D. Nieves Roldán [a]

[a] *Universidad de Murcia, Departamento de Matemáticas, Campus de Espinardo, 30100 Murcia, Spain*
[b] *Universitat Politècnica de Catalunya, Departament de Matemàtiques, Institut de Matemàtiques de la UPC (IMTech), Colom 1, 08222 Terrassa, Spain*

A R T I C L E I N F O

*Article history:*
Received 19 June 2023
Available online 31 October 2023
Submitted by E. Braverman

*Keywords:*
Difference equations
Non-periodic solutions
Accumulation points
Boundedness
Kronecker's theorem
First integral

A B S T R A C T

In this paper, we are interested in analyzing the dynamics of the fourth-order difference equation $x_{n+4} = \max\{x_{n+3}, x_{n+2}, x_{n+1}, 0\} - x_n$, with arbitrary real initial conditions. We fully determine the accumulation point sets of the non-periodic solutions that, in fact, are configured as proper compact intervals of the real line. This study complements the previous knowledge of the dynamics of the difference equation already achieved in Csörnyei and Laczkovich (2001) [5] and Linero Bas and Nieves Roldán (2021) [10].



## 1. Introduction

By a difference equation of max-type, we understand an autonomous or non-autonomous difference equation whose solutions are generated by a recurrence law involving the max operator. There exists a large literature dealing with different classes of max-type equations, in which the main interest is to know the dynamics at large of the orbits generated by the recurrence, and try to apply these equations for the modeling of processes appearing in fields as Biology, Economy, Control Theory,...; for more information, consult books [8], [16] or the survey [11], and references therein.

One of the most popular difference equation of this class is the max-type version of Lyness equation, given by $x_{n+2} = \max\{1, x_{n+1}\}/x_n$, or its generalization in the form $x_{n+2} = \max\{A, x_{n+1}^k\}/(x_{n+1}^\ell x_n)$, where $A, k, \ell$ are real coefficients. In particular, in the case of positive initial conditions, by the change of variables $y_n = \ln x_n$, the difference equation $x_{n+2} = \max\{1, x_{n+1}\}/x_n$ is transformed into $y_{n+2} = \max\{y_{n+1}, 0\} - y_n$;

* Corresponding author.
*E-mail addresses:* lineroba@um.es (A. Linero Bas), victor.manosa@upc.edu (V. Mañosa), daniel.nieves@um.es (D. Nieves Roldán).

https://doi.org/10.1016/j.jmaa.2023.127895
0022-247X/