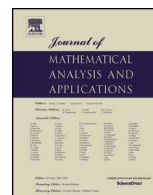




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# On the accumulation points of non-periodic orbits of a difference equation of fourth order

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## ARTICLE INFO

*Article history:*

Received 19 June 2023

Available online 31 October 2023

Submitted by E. Braverman

*Keywords:*

Difference equations

Non-periodic solutions

Accumulation points

Boundedness

Kronecker's theorem

First integral

## ABSTRACT

In this paper, we are interested in analyzing the dynamics of the fourth-order difference equation  $x_{n+4} = \max\{x_{n+3}, x_{n+2}, x_{n+1}, 0\} - x_n$ , with arbitrary real initial conditions. We fully determine the accumulation point sets of the non-periodic solutions that, in fact, are configured as proper compact intervals of the real line. This study complements the previous knowledge of the dynamics of the difference equation already achieved in Csörnyei and Laczkovich (2001) [5] and Linero Bas and Nieves Roldán (2021) [10].

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## 1. Introduction

By a difference equation of max-type, we understand an autonomous or non-autonomous difference equation whose solutions are generated by a recurrence law involving the max operator. There exists a large literature dealing with different classes of max-type equations, in which the main interest is to know the dynamics at large of the orbits generated by the recurrence, and try to apply these equations for the modeling of processes appearing in fields as Biology, Economy, Control Theory,...; for more information, consult books [8], [16] or the survey [11], and references therein.

One of the most popular difference equation of this class is the max-type version of Lyness equation, given by  $x_{n+2} = \max\{1, x_{n+1}\}/x_n$ , or its generalization in the form  $x_{n+2} = \max\{A, x_{n+1}^k\}/(x_{n+1}^\ell x_n)$ , where  $A, k, \ell$  are real coefficients. In particular, in the case of positive initial conditions, by the change of variables  $y_n = \ln x_n$ , the difference equation  $x_{n+2} = \max\{1, x_{n+1}\}/x_n$  is transformed into  $y_{n+2} = \max\{y_{n+1}, 0\} - y_n$ ;

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