Nonlinear Analysis: Hybrid Systems 49 (2023) 101350

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

On the indices of singular points for planar bounded piecewise smooth polynomial vector field

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ARTICLE INFO

Article history:
Received 10 June 2022
Received in revised form 1 December 2022
Accepted 4 March 2023
Available online xxxx

MSC:
34C37
34C07
37G15

Keywords:
Bounded vector field
Index of singular points
Piecewise smooth polynomial vector field
Stereographic projection

ABSTRACT

We prove that for any piecewise-smooth bounded polynomial vector field in $\mathbb{R}^2$ with finitely many finite $\mathcal{H}$-singular points (which include singular points, hyperbolic pseudo-equilibria and two fold singularities), the sum of the indices of all its finite $\mathcal{H}$-singular points is 1.

1. Introduction and statement of the main result

A planar polynomial differential system of the form

$$\begin{cases} \dfrac{dx}{dt} = P(x, y), \\ \dfrac{dy}{dt} = Q(x, y), \end{cases} \tag{1}$$

where $P(x, y)$ and $Q(x, y)$ are polynomials in the variables x and y. System (1) is called a polynomial system of degree m if m is the maximum degree of the polynomials $P(x, y)$ and $Q(x, y)$. We denote $Z(x, y) = (P(x, y), Q(x, y))$ the associated vector field of system (1).

In the qualitative theory of planar polynomial differential systems [1,2], one of the most important problems is the determination and distribution of limit cycles, which is known as the famous Hilbert's 16th problem. Since this problem is very difficult, mathematicians pay attention to the special forms of system (1), for Liénard systems see [3–5], for Z_2-equivariant systems see [6–9], for Hamiltonian systems see [10,11].

Definition 1. A vector field (1) is said bounded when all its orbits are bounded for $t \geqslant 0$.

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https://doi.org/10.1016/j.nahs.2023.101350
1751-570X/