Invariant Tori and Cylinders for a Class of Perturbed Hamiltonian Systems

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Abstract. We start with a relativistic model of the Kepler Problem, which is an isoenergetically non-degenerate central force problem in 2 dimensions. Then we prove the persistence of invariant cylinders and tori for a class of non Hamiltonian perturbations of this system.

§1. Introduction.

Consider the Hamiltonian $\bar{H}_\varepsilon : \mathbb{R}^+ \times S^1 \times \mathbb{R}^2 \to \mathbb{R}$ defined by

$$\bar{H}_\varepsilon(r,\theta,p_r,p_\theta) = \frac{p_r^2}{2} + \frac{p_\theta^2 - 2\varepsilon}{2r^2} - \frac{1}{r}, \tag{1}$$

where $\varepsilon \in \mathbb{R}^+$. Notice that if $\varepsilon = 0$ then $\bar{H}_\varepsilon$ is the Kepler Hamiltonian, and if $\varepsilon \neq 0$ then it is the correction given by special relativity or by a first order approximation in general relativity to the Kepler problem. These Hamiltonians have the general form

$$\frac{p_r^2}{2} + \frac{p_\theta^2}{2r^2} - (\frac{\alpha}{r} + p_0)^2 ,$$

where p_0 is a positive constant of motion and $\alpha > 0$. If $\alpha < 0$, they describe the motion of a particle in the relativistic coulombian electric field, produced by a charged particle of the same sign (see [**T**]).

For the special relativity correction, the coefficients of the terms in r^{-1} and r^{-2} are

$$2\alpha_s p_{0s} = (1 + E/(mc_0^2))\gamma \quad , \quad \alpha_s^2 = \gamma^2/(2mc_0^2)$$

respectively. Here c_0 is the velocity of light, m is the mass of the particle, γ is the universal gravitational constant times the mass of the central body and E is the constant of total energy. A first order approximation to general relativity (Schwarzschild field) considered in [**P**], gives the values

$$2\alpha_g p_{0g} = (1 + 4E/(mc_0^2))\gamma \quad , \quad \alpha_g^2 = 6\alpha_s^2$$

for the above coefficients. Perturbation computations to estimate the precesion of Mercury show that it depends only on the coefficient α_s^2 or α_g^2