# Transversal Ejection–Collision Orbits for the Restricted Problem and the Hill's Problem with Applications\*

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It is shown that the restricted problem and the Hill's problem have transversal ejection—collision orbits. These orbits are used to prove that both problems have no  $C^1$ -extendable regular integrals. © 1988 Academic Press, Inc.

### Introduction

We consider the circular planar restricted three-body problem (usually, the restricted problem) in a rotating coordinate system  $\mathbf{q} = (q_1, q_2)$  of rotational frequency equal to 1. In this frame we locate the larger primary  $m_1$  of mass  $1 - \mu$  at the origin, and the smaller primary  $m_2$  of mass  $\mu$  at the position  $\mathbf{e}_2 = (-1, 0)$ , where  $\mu \in [0, \frac{1}{2}]$ . The Hamiltonian which governs the motion of the zero mass particle  $m_3$  is given by

$$H = |\mathbf{p}|^2 / 2 + q_2 p_1 - q_1 p_2 - |\mathbf{q}|^{-1} + \mu(|\mathbf{q}|^{-1} - |\mathbf{q} - e_2|^{-1} - p_2), \quad (0.1)$$

where  $\mathbf{p} = (p_1, p_2)$  are the momentum variables conjugate to the  $\mathbf{q}$ , and | | denotes the Euclidean norm in  $\mathbb{R}^2$ .

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