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N-Dimensional Zero-Hopf Bifurcation of Polynomial Differential Systems via Averaging Theory of Second Order

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Abstract
Using the averaging theory of second order, we study the limit cycles which bifurcate from a zero-Hopf equilibrium point of polynomial vector fields with cubic nonlinearities in $\mathbb{R}^n$. We prove that there are at least 3^{n-2} limit cycles bifurcating from such zero-Hopf equilibrium points. Moreover, we provide an example in dimension 6 showing that this number of limit cycles is reached.

Keywords Hopf bifurcation · Averaging theory · Cubic polynomial differential systems

Mathematics Subject Classification (2010) 34C07 · 34C05 · 34C40

1 Introduction and Statement of the Main Result

Our goal is to study the limit cycles that bifurcate from a zero-Hopf equilibrium of polynomial differential systems in $\mathbb{R}^n$ with cubic nonlinearities by using the averaging theory.

In [5], the authors studied the Hopf bifurcation in dimension $n > 2$, by using the first-order averaging method. They proved that at least 2^{n-3} limit cycles can bifurcate from one singularity with eigenvalues $\pm bi$ and $n-2$ zeros, i.e., from a *zero-Hopf equilibrium* of $\mathbb{R}^n$. They proved for the first time that the number of bifurcated limit cycles in a Hopf bifurcation

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