



Saddle–node bifurcation of limit cycles in an epidemic model with two levels of awareness

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ABSTRACT

In this paper we study the appearance of bifurcations of limit cycles in an epidemic model with two types of aware individuals. All the transition rates are constant except for the alerting decay rate of the most aware individuals and the rate of creation of the less aware individuals, which depend on the disease prevalence in a non-linear way. For the ODE model, the numerical computation of the limit cycles and the study of their stability are made by means of the Poincaré map. Moreover, sufficient conditions for the existence of an endemic equilibrium are also obtained. These conditions involve a rather natural relationship between the transmissibility of the disease and that of awareness. Finally, stochastic simulations of the model under a very low rate of imported cases are used to confirm the scenarios of bistability (endemic equilibrium and limit cycle) observed in the solutions of the ODE model.

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1. Introduction

The role of human behaviour has been increasingly considered in epidemiological modelling since the early 2000s [1]. The spread of COVID-19 has highlighted even more its important role in the progress of infectious diseases. Besides institutional measures as mobility restrictions, mandatory use of facemasks, or school closings, self-initiated individual behaviours related to risk aversion are recognized as a driving force in epidemic dynamics [2,3]. Another important example of the impact of self-initiated individual behaviours on epidemic spreading is given by the evolution of sexually transmitted diseases (STDs). The current resurgence in the number of cases of STD such as gonorrhea and syphilis began in the mid-1990s, after the striking decline in the number of STD cases following the appearance of the HIV in the early 1980s and the subsequent widespread use of condoms. However, a lower perception of risk following the introduction of antiretroviral therapies for HIV led to a decrease in condom use and the current explosion of STD cases (see [4] and references therein).

One way to model such behavioural changes in deterministic models is to modify the incidence term βSI where β denotes the rate of disease transmission, and S and I are the number of susceptible and infected individuals, respectively. The simplest way to modify it is by assuming that β is no longer constant but

a decreasing function of the prevalence of the disease [3,5–7]. In this mean-field formulation of the incidence term, β depends on the contact rate as well as on the probability of transmission during an infectious contact. So, its reduction can reflect a diminution in the number of social contacts (social distancing), the adoption of measures to prevent infection while keeping the same contact rate (decrease of the infection probability), or both.

On the other hand, it is well known that the perception of infection risk is uneven among susceptible individuals [8]. One way to introduce some heterogeneity in risk-taking propensity has been to include more types of uninfected individuals characterized by their level of responsiveness to risk. For instance, the Susceptible–Aware–Infectious–Susceptible (SAIS) model considers a new class of non-infected individuals with a higher risk aversion than the susceptible ones, the so-called aware or alerted individuals, who are characterized by a lower transmission rate [9].

A basic ingredient in such a modelling approach is the transmission of awareness among individuals [10]. In [4] the authors considered an SAIS model where alerted individuals were able to transmit awareness by convincing non-aware individuals to take preventive measures against the infection, which is an example of self-initiated individual behaviour. Moreover, a new class of aware individuals, the so-called unwilling (U) individuals, is also introduced. They are characterized by a lower level of alertness which is translated into a lack of willingness to transmit awareness to susceptible individuals. The existence of this second class of aware individuals turns out to be necessary to have oscillatory solutions of the SAUIS model with no births and deaths in the population.

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