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Limit Cycles of a Class of Discontinuous Piecewise Differential Systems Separated by the Curve $y = x^n$ Via Averaging Theory

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Recently there is increasing interest in studying the limit cycles of the piecewise differential systems due to their many applications. In this paper we prove that the linear system $\dot{x} = y$, $\dot{y} = -x$, can produce at most seven crossing limit cycles for $n \geq 4$ using the averaging theory of first order, where the bounds ≤ 4 for $n \geq 4$ even and the bounds ≤ 7 for $n \geq 5$ odd are reachable, when it is perturbed by discontinuous piecewise polynomials formed by two pieces separated by the curve $y = x^n$ $(n \geq 4)$, and having in each piece a quadratic polynomial differential system. Using the averaging theory of second order the perturbed system can be chosen in such way that it has $0, 1, 2, 3, 4, 5$ or 6 crossing limit cycles for $n \geq 4$ even and, furthermore, under a particular condition we prove that the number of crossing limit cycles does not exceed 9 (resp., 11) for $4 \leq n \leq 74$ even (resp., $n \geq 76$ even). The averaging theory of second order produces the same number of crossing limit cycles as the averaging theory of first order if $n \geq 5$ is odd. The main tools for proving our results are the new averaging theory developed for studying the crossing limit cycles of the discontinuous piecewise differential systems, and the theory for studying the zeros of a function using the extended Chebyshev systems.

Keywords: Limit cycles; the method of averaging; discontinuous piecewise differential systems.

1. Introduction and Statement of the Main Results

The study of the limit cycles of planar differential systems is one of the main problems in the qualitative theory of differential systems. The method of averaging has been one of the effectively analytical methods to detect the existence of limit cycles of the nonlinear differential equations. For the smooth differential equations the averaging theory can be found in some monographs [Hale, 1980; Guckenheimer & Holmes, 1983; Sanders *et al.*, 2007]. In recent years the classical averaging theory for computing periodic solutions was developed rapidly [Buică & Llibre, 2004; Llibre *et al.*, 2014; Llibre *et al.*, 2015b; Giné *et al.*, 2013; Giné *et al.*, 2016; Llibre & Novaes, 2015; Novaes & Silva, 2021]. Recently the averaging theory for computing periodic solutions has been developed for discontinuous