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A CLASSIFICATION OF BRAID TYPES FOR PERIODIC ORBITS OF DIFFEOMORPHISMS OF SURFACES OF GENUS ONE WITH TOPOLOGICAL ENTROPY ZERO

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Abstract

We classify the braid types that can occur for finite unions of periodic orbits of diffeomorphisms of surfaces of genus one with zero topological entropy.

1. Introduction

In this paper we classify the braid types that can occur for finite unions of periodic orbits of diffeomorphisms of surfaces of genus one with zero topological entropy. This extends the analysis from the case of genus zero [**LM1**]. The case of most interest to us is diffeomorphisms of the torus, isotopic to the identity. This is relevant to the behaviour of three coupled oscillators, for example. A good picture of their dynamics is developing [**KMG**], [**LM2**],[**MZ**], [**H2**], [**F**], [**BGKM**]. We hope that our results will help solve the intriguing problem of understanding the boundary of zero topological entropy in the space of C^1 diffeomorphisms of the torus.

We begin by establishing some notation and recalling the definition of braid type. Let $f : X \to X$ be a diffeomorphism of an oriented manifold X. Write $h(f)$ for the topological entropy of f. We write $o-p$, $o-r$ for orientation-preserving and reversing, respectively. Given two diffeomorphisms $f : X \to X$, $g : Y \to Y$ of oriented manifolds, we write $f \simeq g$ if there exists an $o-p$ conjugacy between them.

Let M be a surface, i.e. a compact connected oriented 2-manifold. Let $f : M \to M$ be a diffeomorphism of M and let $\mathcal{P}$ be a finite union of periodic orbits for f. Then we define $f_{\mathcal{P}} : M_{\mathcal{P}} \to M_{\mathcal{P}}$ by removing $\mathcal{P}$ from M and recompactifying by replacing the points of $\mathcal{P}$ by circles on which $f_{\mathcal{P}}$ is the projective action of $Df[B]$.