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ON THE OPTIMAL STATION KEEPING CONTROL OF HALO ORBITS†

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Abstract—Techniques for computing and controlling a halo orbit are considered in this paper. It presents a semi-analytical theory for the halo orbits in the Restricted Three Bodies Problem (RTBP), that is valid and amenable to computation to any order. Results are presented up to order 11. The Floquet modes of the monodromy matrix are used to define a local optimal control procedure through the concepts of projection and gain functions.

1. INTRODUCTION

The three bodies problem has occupied a central place in celestial mechanics and analytical dynamics through more than two centuries, where it has served as a rich field to develop and test new theories. The great interest in space sciences of the last decades has renewed the research activities on the subject and, in particular, in the Restricted Three Bodies Problem, RBTP. It is well known that the RTBP describes the motion of a particle of very small mass, subject to the gravitational attraction of two massive bodies that are in circular motion around their centre of masses. Since the times of Euler and Lagrange it is known that this dynamical system has five libration points. A particle placed initially in one of them will remain in it forever.

The solar system offers a few cases of application of the three bodies problem, where the real world problem can be considered as a perturbation of the ideal RTBP. For example, in the Sun and Earth-Moon barycentre system a first approach will be to consider the Earth–Moon barycentre moving in a circular motion around the Sun and with an angular velocity equal to the mean motion of the barycentre longitude at a given epoch. The distance of the barycentre to the Sun is taken as the mean distance and the mass of the Sun is slightly changed in order to satisfy Kepler's law.

The effects of: (a) a non-circular motion of the barycentre; (b) the Earth-Moon system; (c) all other planets;(d) solar radiation pressure, are considered as small perturbing effects. In the complex field of forces of the real world the dynamical libration points do not subsist. Nevertheless, it is possible to introduce geometrical libration points that are given by the same mathematical relations being used in the RTBP. As the perturbations of the real world are not too big, those formal libration points retain some properties of the original libration points of the RTBP. In particular, a particle placed on one of them moves slowly around it provided it is not too far from them. Therefore the libration points can be used to place a spacecraft that will remain in a fixed configuration[1–4]. The libration points L1 and L2 are in the line joining the two primary bodies are near the small one—a real good place for an observations or communications type of space mission. The exact geometrical point may present problems with the radio electrical communications, but fortunately, there are orbits such that the motion stays near the libration point not being colinear with the primary bodies.

For systems like the Earth–Moon–Sun or Earth–Moon, the motion near the corresponding L1 or L2 point is very unstable and the spacecraft has to be manoeuvred to keep it near the libration point. The American ISEE-C mission has already been maintained in such an orbit for a period of 4 years. The scientific SOHO mission proposed by the European Space Agency, will be placed near the L1 point too, and because of scientific requirements, the manoeuvres needed for orbit maintenance have to be as small as possible. New techniques for orbit maintenance have been developed to reduce the total delta-V requirement to a minimum value.

Reference[5] contains the full details of a study addressing all aspects of the station keeping of orbits near the libration points. The present paper presents the fundamental ideas applied to the RTBP without

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