CAPTURE/ESCAPE BOUNDARY IN THE COLLINEAR RESTRICTED THREE-BODY PROBLEM

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ABSTRACT. We study the cantorian structure of the successive intersections of the invariant manifolds of infinity (parabolic orbits) with a certain surface of section. The first of these intersections is computed numerically. The structure of the set of orbits of capture or escape after n binary collisions is given.

1. FORMULATION OF THE PROBLEM. EQUATIONS OF MOTION.

Two bodies (called primaries) of point masses m_1 and m_2 are moving in an elliptic collision orbit under the influence of their mutual gravitational attraction and a third body of mass $m_3 \approx 0$ (attracted by the previous two but not influencing their motion) moves in the line defined by the two primaries. The collinear restricted three body problem is to describe the motion of this third body.

We select units of length, time and mass such that the length of the major axis of the collision elliptic orbit equals 2, the period is 2π and $m_1 = m$, $m_2 = 1 - m$ with $m \in (0,1)$. Units are taken in such a way that the gravitation constant equals 1. We take the center of masses at the origin.

Let $-x_1, x_2$ ($x_i \geqslant 0$) be the coordinates of m_1, m_2, respectively. Then the motion of the two primaries is given by

$$x_1 = (1-m)(1-\cos E) \;, \qquad x_2 = m(1-\cos E) \;, \tag{1}$$

with $t = E - \sin E$. The parameter E is the so called eccentric anomaly and the origin of time is taken at a collision between m_1 and m_2. We remark that t, E are defined modulus 2π and from now on this will be understood without explicit mention each time that t or E appears.

Let $x \geqslant 0$ be the coordinate of the third body. We assume $x \geqslant x_2$ (see Figure 1). The equation of motion of the third body is:

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