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Research article

Time-reversibility and integrability of $p : -q$ resonant vector fields

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Abstract: We study the local analytical integrability in a neighborhood of $p : -q$ resonant singular point of a two-dimensional vector field and its connection to time-reversibility with respect to the non-smooth involution $\varphi(x, y) = (y^{p/q}, x^{q/p})$. Some generalizations of the theory developed by Sibirsky for the $1 : -1$ resonant case to the $p : -q$ resonant case are presented.

Keywords: planar systems of ODEs; time-reversibility; integrability; resonant singularity
Mathematics Subject Classification: 34C14, 37C79

1. Introduction

Consider an n-dimensional system of ordinary differential equations

$$\dot{x} = F(x), \tag{1.1}$$

where $F(x)$ is an n-dimensional vector-function defined on some domain Ω of $\mathbb{R}^n$ or $\mathbb{C}^n$. It is said (see e.g., [2, 15]) that system (1.1) is *time-reversible* on Ω if there exists an involution ψ defined on Ω such