



# Dynamics and Darboux Integrability of the $D_2$ Polynomial Vector Fields of Degree 2 in $\mathbb{R}^3$

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## Abstract

We characterize the Darboux integrability and the global dynamics of the 3-dimensional polynomial differential systems of degree 2 which are invariant under the  $D_2$  symmetric group, which have been partially studied previously by several authors.

**Keywords** Integrability · Darboux invariant · Poincaré sphere · Global dynamics ·  $D_2$  symmetric polynomial vector fields

**Mathematics Subject Classification (2010)** 34C05 · 34C07 · 34C08

## 1 Introduction and Statement of the Results

Differential systems having some symmetries appear often in many applications, and consequently have been studied by several authors, see for instance [3, 6, 8, 9, 14].

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