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Criteria for the nonexistence of periodic orbits in planar differential systems

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Abstract. In this work we summarize some well-known criteria for the nonexistence of periodic orbits in planar differential systems. Additionally we present two new criteria and illustrate with examples these criteria.

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1 Introduction and statement of the main results

We consider a *planar differential system* that we write as

$$\frac{dx}{dt} = \dot{x} = P(x,y), \qquad \frac{dy}{dt} = \dot{y} = Q(x,y), \tag{1}$$

where $P(x,y)$ and $Q(x,y)$ are C^1 real functions in the variables x and y, and t is the independent variable.

The objective of this note is double, first we recall the more well-known results for the nonexistence of periodic orbits of a differential system (1). Second we provide two new criteria for the nonexistence of periodic orbits of system (1).

As far as we know one of the first criterium of nonexistence is the following one due to Poincaré.

Theorem 1 (Poincaré Method of Tangential Curves)**.** *Consider a family of curves $F(x,y) = C$, where $F(x,y)$ is continuously differentiable. If in a region R the quantity*

$$\frac{dF}{dt} = P\frac{\partial F}{\partial x} + Q\frac{\partial F}{\partial y}$$

has constant sign, and the curve

$$P\frac{\partial F}{\partial x} + Q\frac{\partial F}{\partial y} = 0$$

(which represents the locus of points of contact between curves in the family and the trajectories of (1)*, and is called a tangential curve) does not contain a whole trajectory of* (1) *or any closed branch, then system* (1) *does not possess a periodic orbit which is entirely contained in R.*

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