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A family of periodic orbits for the extended Hamiltonian system of the Van der Pol oscillator

Jean-Marc Ginoux [a], Jaume Llibre [b,*]

[a] Aix Marseille Univ, Université de Toulon, CNRS, CPT, Marseille, France
[b] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain

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ABSTRACT

In this paper we show a new way of using the averaging theory for studying families of periodic orbits of a Hamiltonian system. We do this study computing a new family of periodic orbits of the extension of the Van der Pol oscillator to a Hamiltonian system of two degrees of freedom.

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1. Introduction and statement of the main results

The classical Van der Pol oscillator is modeled by the second-order differential equation

$$\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0, \tag{1}$$

where $x = x(t)$ is the position coordinate at time t, μ is a parameter, and the dot denotes derivative with respect to the time t, see [9–11].

Initially the differential equation (1) allowed to Van der Pol to explain the stable oscillations observed in electrical circuits employing vacuum tubes. Later on this differential equation has been used for explaining different phenomena in biology, physics, ... Thus in biology was utilized as a model for studying the action potentials of neurons, see for instance [3,4,7]. While in physics equation (1) was also used in phonation to model the right and left vocal fold oscillators (see [6]), and in seismology to model two plates in a geological fault (see [1]), ...

More recently in 2015, see equations (9) of [8], the Van der Pol oscillator was written in the Hamiltonian formalism by extending it to a four-dimensional autonomous differential as follows

$$\begin{aligned} &\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0, \\ &\ddot{y} + \mu(1 - x^2)\dot{y} + y = 0. \end{aligned} \tag{2}$$

* Corresponding author.
E-mail addresses: ginoux@univ-tln.fr (J.-M. Ginoux), jllibre@mat.uab.cat (J. Llibre).

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