International Journal of Bifurcation and Chaos, Vol. 32, No. 1 (2022) 2250003 (28 pages)

DOI: 10.1142/S0218127422500031

Limit Cycles of Continuous Piecewise Differential Systems Formed by Linear and Quadratic Isochronous Centers I

Bilal Ghermoul[*,‡], Jaume Llibre[†,§] and Tayeb Salhi[*,¶]

[*]*Department of Mathematics,*
University Mohamed El Bachir El Ibrahimi,
Bordj Bou Arreridj 34265, El–Anasser, Algeria

[†]*Departament de Matemàtiques,*
Universitat Autònoma de Barcelona,
08193 Bellaterra, Barcelona, Catalonia, Spain
[‡]*bilal.ghermoul@univ-bba.dz*
[§]*jllibre@mat.uab.cat*
[¶]*t.salhi@univ-bba.dz*

Received October 17, 2020; Revised December 23, 2020

First, we study the planar continuous piecewise differential systems separated by the straight line $x = 0$ formed by a linear isochronous center in $x > 0$ and an isochronous quadratic center in $x < 0$. We prove that these piecewise differential systems cannot have crossing periodic orbits, and consequently they do not have crossing limit cycles.

Second, we study the crossing periodic orbits and limit cycles of the planar continuous piecewise differential systems separated by the straight line $x = 0$ having in $x > 0$ the general quadratic isochronous center $\dot{x} = -y + x^2 - y^2$, $\dot{y} = x(1 + 2y)$ after an affine transformation, and in $x < 0$ an arbitrary quadratic isochronous center. For these kind of continuous piecewise differential systems the maximum number of crossing limit cycles is one, and there are examples having one crossing limit cycles.

In short for these families of continuous piecewise differential systems we have solved the extension of the 16th Hilbert problem.

Keywords: Limit cycles; isochronous quadratic centers; continuous piecewise linear differential systems; first integrals.

1. Introduction

In the qualitative theory of planar differential systems, a *limit cycle* is an isolated periodic solution in the set of all periodic solutions, which remained the most sought solutions when modeling physical systems in the plane. As far as we know, the notion of limit cycle appeared in the year 1885 in the work of Poincaré [1928].

Most of the early examples in the theory of limit cycles in planar differential systems were commonly related to practical problems with mechanical and electronic systems, but periodic behavior appears in all branches of the sciences. To determine the existence or nonexistence of limit cycles is one of the more difficult objects in the qualitative theory of planar differential equations. A large amount of