ON THE BASIN OF ATTRACTION OF DISSIPATIVE PLANAR VECTOR FIELDS

A. GASULL[1] AND J. SOTOMAYOR[2]

Abstract. An estimate for the size of the basin of attraction of an equilibrium point for a class of planar dissipative vector fields is given here. Our main result, which generalizes a theorem of Krasowskii, is applied to give several sufficient conditions for global asymptotic stability.

1. INTRODUCTION.

Consider the (x, y)-plane $\mathbf{R}^2$, endowed with the Euclidean norm $|p| = (x^2 + y^2)^{1/2}$, $p = (x, y)$.

A C^1 planar vector field $V = X\frac{\partial}{\partial x} + Y\frac{\partial}{\partial y}$, is called *dissipative* on a region D if $divV = \frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} \leq 0$ on D and the equality only holds on a set with Lebesgue measure zero. If the equality never holds on D we will say that V is *strictly dissipative* on D.

This paper is concerned with the estimate of the size of the *basin of attraction* A_V of an equilibrium point p of a C^1 vector field V. The aim is to determine a disk, as large as posible, centered at p, contained in A_V, i.e. such that the orbit through any of its points tends to p as the time goes to infinity. Conditions on V for $A_V = \mathbf{R}^2$ (global asymptotic stability) have been studied in [**GLS**], where references for the bakground of this problem can be found. The present paper focuses on situations not necessarily covered in [**GLS**], as discussed in Section 4.

Define the following functions associated to a planar vector field W

$$m(r) = \min\{|W(p)|;\ |p| = r\},$$
$$M(r) = \max\{|W(p)|;\ |p| = r\}.$$

For $0 < R \leq S \leq \infty$ define

$$L_S(R) = \frac{1}{2\pi}\int_R^S m(r)\, dr.$$

This is a non increasing function when $S < \infty$ or when $\int_R^\infty m(r)\, dr < \infty$, otherwise it is constant equal to infinity.

If B is a positive C^1 function such that $B \cdot W$ is (strictly) dissipative on D we will say that B is a *(strict) Dulac function* for W on D.

The appropiate choice of Dulac functions has been shown to actually enlarge the domain where a vector field is dissipative.

Call $T = \sup\{r > 0;\ L_S(r) - rM(r) \geq 0\}$ where the functions L_S and M are associated to $B \cdot V$; $T = \infty$ if this supremum is infinity. Note that always $0 < T \leq S$.

The main result of this paper is the following Theorem.

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[1] Partially supported by CIGYT grant number PB86-0351.

[2] Current address : IHES, 35 Route de Chartres, Bures sur Yvette, 91440 FRANCE