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Limit Cycles of Vector Fields of the Form $X(v) = Av + f(v)\, Bv$

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1. INTRODUCTION

In this paper we study the phase portraits of planar vector fields X of the form

$$X(v) = Av + f(v)\, Bv, \tag{1}$$

where A and B are 2×2 matrices, $\det A \neq 0$ and $f\colon \mathbb{R}^2 \to \mathbb{R}$ is a smooth real function such that its expression in polar coordinates is $f(r\cos\theta, r\sin\theta) = r^D \bar{f}(\theta)$ with $D \geqslant 1$ (note that if f is a homogeneous function then $\bar{f}(\theta) = f(\cos\theta, \sin\theta)$). In this case we shall say that f is a *homogeneous function of degree D*. If f is such that $f(\lambda x, \lambda y) = \lambda^D f(x, y)$ we shall say that f is *homogeneous in the usual sense*. This class of vector fields have been studied by C. Chicone [1] as an important extension of a less general class of quadratic vector fields considered by D. E. Koditschek and K. S. Narendra [3, 4]. There are two hypotheses H_i $(i = 1, 2)$, one for the matrices A and B, the other for the function f.

For a 2×2 matrix C let C^t denote the transpose of C. Then, the symmetric part of C is given by $(C)_s = \frac{1}{2}(C + C^t)$. If J is the sympletic 2×2 matrix $\left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$, then the *hypothesis* H_1 states that $(JB)_s$ and $(B^tJA)_s$ are definite and have the same sign. Note that if these two matrices associated to X are definite with opposite sign, then the system $-X$ satisfies hypothesis H_1.

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