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CHORDAL QUADRATIC SYSTEMS*

ARMENGOL GASULL, SHENG LI-REN AND JAUME LLIBRE

ABSTRACT. A quadratic system is called chordal if all its singularities are on the equator of the Poincaré sphere. First, we establish necessary and sufficient conditions for a quadratic system to be chordal. Later, we determine all the phase portraits for such systems.

1. Introduction. This paper contains a study of those two-dimensional autonomous systems with quadratic polynomial right-hand sides without finite singularities. Such systems will be referred to as chordal quadratic systems (CQS, for abbreviation). The chordal systems were studied by Kaplan, see [6] and [7]. The name of chordal system is due to the fact that a such system has all its solutions starting and ending at the equator of the Poincaré sphere. For a survey on quadratic systems (QS, for abbreviation), see Coppel [4] and Ye Yanqian [13]. At the end of the paper [4], Coppel states that what remains to be done for quadratic systems is to determine all possible phase portraits and, ideally, to characterize them by means of algebraic inequalities on the coefficients.

This paper first establishes necessary and sufficient conditions for a QS to have all its singularities at infinity (on the equator of the Poincaré sphere), i.e., to be a CQS, and then determines all possible phase portraits for such CQS.

Our main result is the following theorem.

THEOREM. *The phase portrait of a chordal quadratic system is homeomorphic (except for perhaps the orientation) to one of the separatrix configurations shown in Figure 1. Furthermore, all the separatrix configurations of Figure 1 are realizable for the chordal quadratic systems.*

REMARK 1. The Figures 1.1 to 1.21 are realizable for properly chordal

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