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Counting configurations of limit cycles and centers

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Abstract. We present several results on the determination of the number and distribution of limit cycles or centers for planar systems of differential equations. In most cases, the study of a recurrence is one of the key points of our approach. These results include the counting of the number of configurations of stabilities of nested limit cycles, the study of the number of different configurations of a given number of limit cycles, the proof of some quadratic lower bounds for Hilbert numbers and some questions about the number of centers for planar polynomial vector fields.

Mathematics subject classification: 34C05, 34C07, 37C27.
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Dedicated to the memory of Professor Constantin Sibirschi

1 Introduction

In the qualitative study of differential systems in the plane, there are several questions about topological configurations that naturally lead to enumeration and combinatorial problems which, sometimes, can be approached by using recurrences.

This paper is devoted to study such type of questions for smooth planar differential systems,

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y). \tag{1}$$

In some parts of the work, the functions P and Q defining these differential equations will also be assumed to be polynomials.

After giving some definitions in Section 2, in Section 3 we will consider the growth of the number of stability configurations of n nested limit cycles of (1) in terms of their stability. We will show that this number of configurations can be explicitly determined using expressions involving the Fibonacci numbers, $F_0 = F_1 = 1$ and $F_{n+2} = F_{n+1} + F_n$. Although it may not be necessary to highlight the importance and ubiquity of the Fibonacci sequence both in arithmetic and geometry problems, and in some applied questions, we point out a couple of examples: their use in graph theory ([33]) and their earliest appearance in Indian mathematics to count the number of sums of 1 and 2 (taking into account the order of the addends) that add up to n. For instance, if $n = 4$, there appear $F_{n+1} = F_{4+1} = 5$ possibilities:

$$1 + 1 + 1 + 1, \; 2 + 1 + 1, \; 1 + 2 + 1, \; 1 + 1 + 2, \; 2 + 2.$$

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