NON-EXISTENCE OF LIMIT CYCLES FOR SOME PREDATOR-PREY SYSTEMS

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Abstract. We give a criterion of non-existence of limit cycles for a type of predator-prey systems. It reduces the problem to the study of the solutions of a nonlinear system of equations. The proof of the criterion is based on the Filippov transformation.

1. INTRODUCTION.

Lotka and Volterra (see [Lo], [V]) proposed a first model for deterministic predator-prey systems. It approximates the global behavior of such systems taking into account a few constant factors of the realistic models (birth rate of the prey, death rate of the predator and the interaction between the species). It has been studied and generalized by many authors (see [G], [K]) in order to get models describing more accurately the most important biological features. One of these improvals, due to Gause, is (see [Hu]):

$$\text{(1)} \qquad \begin{cases} \frac{dx}{dt} = b(x)\left(F(x) - \varphi(y)\right), \\ \frac{dy}{dt} = a(y)g(x). \end{cases}$$

Our purpose is to give a criterion for the non-existence of limit cycles for systems of the type (1).

2. MAIN RESULT.

Note that Liénard systems, see [L], can be thought as a particular case of (1), in which $b(x) \equiv 1$, $a(y) \equiv 1$, and $\varphi(y) = y$. The Filippov transformation, see [F], is applied to study Liénard systems with $xg(x) > 0$, by doing the change of variables $z = \int_0^x g(s)\, ds$. This change is well-defined between $\Omega^- = \{(x,y) : x < 0\}$ and $R = \{(z,y) : z > 0\}$, and between $\Omega^+ = \{(x,y) : x > 0\}$ and R. Its main idea is to unfold the system in two different systems according to a reflection of the part of the trajectories lying in Ω^- on the righthand side, and a deformation by the function $g(x)$. So, it allows to compare both sides of a periodic orbit and gives information about it.

The generalization of the Filippov transformation that we present can be applied to Gause systems of type (1).

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