



Regularization of Brouwer Translation Theorem

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Abstract

The celebrated Brouwer translation theorem asserts that for a preserving orientation fixed point free homeomorphism of the plane, each point belongs to an invariant region where the dynamics is continuously conjugate to a translation. In this work we prove that if we start with a C^m , $m \in \mathbb{N} \cup \{\infty\}$, diffeomorphism then the referred conjugacy has the same kind of regularity.

Keywords Brouwer translation theorem · Smooth conjugation · Homeomorphism · Diffeomorphism

Mathematics Subject Classification Primary 37C15; Secondary 54H20

Introduction and Main Result

Let F be an orientation-preserving homeomorphism of \mathbb{R}^2 which is fixed point free. The *Brouwer's plane translation theorem* asserts that every $z \in \mathbb{R}^2$ is contained in a domain of translation \mathcal{U} for F , that is an open connected subset of \mathbb{R}^2 , whose boundary is $L \cup F(L)$ where L is the image of a proper embedding of \mathbb{R} in \mathbb{R}^2 , such that L separates $F(L)$ and $F^{-1}(L)$ (L is called a Brouwer line). Moreover,

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