Qualitative Theory of Dynamical Systems (2023) 22:44
https://doi.org/10.1007/s12346-023-00746-7

Number of Limit Cycles for Planar Systems with Invariant Algebraic Curves

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Received: 27 October 2022 / Accepted: 14 January 2023

Abstract
For planar polynomials systems the existence of an invariant algebraic curve limits the number of limit cycles not contained in this curve. We present a general approach to prove non-existence of periodic orbits not contained in this given algebraic curve. When the method is applied to parametric families of polynomial systems that have limit cycles for some values of the parameters, our result leads to effective algebraic conditions on the parameters that force non-existence of the periodic orbits. As applications we consider several families of quadratic systems: the ones having some quadratic invariant algebraic curve, the known ones having an algebraic limit cycle, a family having a cubic invariant algebraic curve and other ones. For any quadratic system with two invariant algebraic curves we prove a finiteness result for its number of limit cycles that only depends on the degrees of these curves. We also consider some families of cubic systems having either a quadratic or a cubic invariant algebraic curve and a family of Liénard systems. We also give a new and simple proof of the known fact that quadratic systems with an invariant parabola have at most one limit cycle. In fact, what we show is that this result is a consequence of the similar result for quadratic systems with an invariant straight line.

Keywords Invariant algebraic curve · Limit cycle · Periodic orbit · Quadratic system · Cubic system · Liénard system

Mathematics Subject Classification Primary 34C07; Secondary 37C27

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Published online: 03 February 2023