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Some open problems in low dimensional dynamical systems

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Abstract
The aim of this paper is to share with the mathematical community a list of 33 problems that I have found along the years in my research. I believe that it is worth to think about them and, hopefully, solve some of the problems or make some substantial progress. Many of them are about planar differential equations but there are also questions about other mathematical aspects: Abel differential equations, difference equations, global asymptotic stability, geometrical questions, problems involving polynomials and some recreational problems with a dynamical component.

Keywords Limit cycle · Period function · Center · Abel differential equation · Piecewise linear differential equation · Global asymptotic stability · Fewnomials · Conjectures · Open problems

Mathematics Subject Classification 34C07 · 37C27 · 34D45 · 37G35 · 13P15

1 Introduction

There are several famous well-known conjectures and open problems, such as the Jacobian conjecture, the Riemann's conjecture, the $3x + 1$ conjecture or Collatz problem, the Goldbach's conjecture, or Hilbert XVI problem, that almost all mathematicians know. Also a very interesting list of 18 open problems, covering many different branches of mathematics, has been published by Smale, see [122]. The aim of this work is much more modest. I will list several concrete problems that I have found along the years. I hope that, at least for some of them, it is possible either to solve or to make some substantial progress towards their solution.

The problems will be classified in seven categories: periodic orbits, period function, piecewise linear systems, Markus–Yamabe and La Salle problems, geometrical problems, questions involving polynomials, and recreational questions with a dynamical flavour. Next

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